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RIEMANN SURFACES AND THEIR PHYSICAL INTERPRETATION

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ABSTRACT

Riemannian spaces are presented in the article as logical structures reflecting physical reality. Presented the modeling of the process of electrification of the Earth as a theory of analytic functions on the Riemann surface in the interpretation of Helmholtz-Monastyrsky, as well as modeling of the Bose-Einstein condensate using the zeta functionin as interpreted by Prof. Grant N. Remmen.

KEYWORDS: *Riemanns Space, Analytical Functions, Dzet Function, Condensate Bose Einstein, Electrification*

1. INTRODUCTION

A deep understanding of the differences between geometry as a logical structure and reflection of physical reality Science is obliged to the works of Bernhard Riemann. Considering a purely mathematical problem of determining the n-dimensional space with a given metric, it received three types of Riemann's spaces, each of which is associated with its classical geometry:

- with the space of positive curvature - elliptical Riemannian geometry;
- with zero curvature space - Euclid's geometry;
- With the space of negative curvature - Lobachevski geometry.

Riemann's spaces are in which the distance between two dots the manifold is determined without reference to the space in which it is located. This is an internal determination of the distance and sets the metric on the manifold [1].

The space of constant curvature occurs at Riemann from the physical requirement so that the "figures" could move in them without "stretching" and "compression", i.e. So that the figures could move like hard bodies and retained their forms while driving. Physical intuition, with which Riemann sets the task for geometry to describe a physically real space, is striking with its prophetic vision. He writes: "The question of whether the assumptions of geometry are valid in the infinitesimal is closely related to the question of the internal cause of the emergence of metric relations in space. In the case of a discrete manifold, the principle of metric relations is already contained in the very concept of manifold, whereas in the case of a continuous manifold it should be searched somewhere else. It follows from this that or the real one that creates the idea of space, forms a discrete variety, or you need to try to explain the occurrence of metric relations in the case of a continuous manifold by the recognition of another factor - because there are communication forces acting on this is real.".

Thus, he confirms the idea of Kant on the "relativity" of the space and the formation of the Metric of space with real physical forces, whether the laws of Cavendish and Coulomb in the three-dimensional space of Euclide, as well as geometric gravity in Einstein, or gravitational fields in quantum gravity.

It is characteristic that for Physicists, Riemann was completely convincing, which cannot be said about mathematicians. For them, the dissertation of Riemann, according to the expression of Professors Zommerfeld, was the "book for seven seals". M.I. Monastyrsky in his book "Bernhard Rimann" says: "There is a remarkable interpretation of the theory of functions, leading its beginning from Helmholtz, which explains the basis for the confidence physicists in the justice of Riemann's results" [2].

2. MODELING THE PROCESS OF ELECTRIFICATION OF THE GLOBE AS THE THEORY OF ANALYTIC FUNCTIONS ON THE RIEMANN SURFACE IN THE INTERPRETATION OF THE HELMHOLTZ - MONASTYRSKY

As interpreted by Helmholtz-Monastyrsky [2], the theory of analytic functions on the Riemann surface we can present as an issue of physics. The Earth has an electric charge, which, because of the Coulomb repulsion, tends to a surface of the planet. The electrification process of the near-Earth environment that behaves like the incompressible fluid looks like according to an expression by N. Tesla a yield state. At that, the energy is primarily transmitted along the curve - the shortest way between a source and a receiver on the Earth's surface. Distribution of currents of the "electric fluid" on the Earth's surface in the well-known project "Global System of Wireless Energy Transmission" by N. Tesla (1904) [3] one describe analytically with the theory of the stationary, two-dimensional, ideal incompressible fluid on the Riemann surface and computer simulation of this process can be carried out. Let us consider a stationary fluid flow u on the plane *(x, y)*.The flow speed at each point has *x*-component *P (x, y)* and *y*-component *Q (x, y)*.Through the cell with sides *Δ x, Δy* per a time unit the mass of liquid outflows (liquid density is constant and equals

$$\int_0^{\Delta y}.\{P\ (x+\ \Delta\ x,\ y+h)-\ P\ (\ y+hx,\}\,d\,h+ \quad (1)$$

$$+\int_0^{\Delta x}.\{Q\ (x+\ l,\ y+\Delta y)-\ Q\ (x+l,y\}\ d\ l$$

Approximating an arbitrary domain Ω with rectangles and applying the Green's formula, we obtain that the integral (1) is equal to:

$$\iint\left(\frac{dP}{dx}+\frac{dQ}{dy}\right)\ dxdy. \quad (2)$$

Since the fluid is incompressible and nowhere appears and disappears in the Ω domain, it follows that the expression (2) is zero. The stronger statement is also reasonable, i.e. flow divergence u is zero:

$$\text{Div U}=\frac{d\,P}{dx}+\frac{dQ}{dy}=0 \quad (3)$$

The flow circulation along the curve C is defined as the integral $\int Pdx+Qdy$.

If this integral along any closed curve is zero, then the flow is called irrotational. For any single-bound domain, it follows that statement *Pdx + Qdy* is a complete differential of the function *u (x, y)*.This function is harmonic.

The function *U (x, y)* is called the flow speed potential. Helmholtz introduced this concept. Curves *U (x, y) = const* are called equipotential lines. A tangent line to the equipotential line forms such an angle α with the axis *x*, that

$$tg\ \alpha = -\frac{d\,U}{dx} / \frac{d\,U}{dy}$$, if only ΔU≠0.

The flow speed vector makes an angle β with the x axis,

$$tg\ \beta = \frac{d\,U}{dy} / \frac{d\,U}{dx},$$

I.e. the flow goes orthogonally to equipotential lines in the direction of increasing U function.

As we remember, the harmonic function u (x, y) defines the function of

$$f(z) = u + i\upsilon$$

Where υ is a conjugate to the harmonic function *u*, defined from Cauchy-Riemann equations (A.4) . Essentially, Cauchy solves the following problem, under which conditions for the complex function *f (z)* the integral ∫*f (z) dz* in a closed loop ℓ is zero. However, he does not speak explicitly of the complex function, but applying pairs of real functions *P(x, y) and Q (x, y),* gets his main result: the integral ∫*f (z) dz* does not depend on the integration path if such conditions are met:

$$\frac{dP}{dx} = \frac{dQ}{dy}; \frac{dP}{dy} = -dQ/dx \tag{4}$$

This condition is a (4) characteristic property of analyticity (holomorphicity) of function of a complex variable. In modern literature, the common name is the Cauchy-Riemann condition. The function *f (z)* people call the complex potential of the flow.

The tangent line to the curve υ = const makes an angle γ with the x axis and

$$tg\ \gamma = -\frac{d\upsilon}{dx} / \frac{d\upsilon}{dy} = \frac{du}{dy} / \frac{du}{dx} = tg\ \beta$$

I.e. the *u* flow goes along the curve *υ = const*. These curves people call streamlines.

The condition $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} \neq 0$, equivalent to *f '(z) ≠ 0,*

Indicates that streamlines are orthogonal to equipotential lines except at points where *f' (z) = 0*

This physical analogy allows us to interpret any properties of analytic functions exceptionally clearly. For example, if the analytic function *f (z)* has at the point z_0 $f'(z_0) = 0$, then curves *u = const* and *υ = const* do not cross at $z_0 = x_0 + iy_0$ at right angles. Such points are called stationary points, e.g. for the function

$$f(z) = a_0 + a2\ z^2$$

curves *u = const* and *υ = const* intersect at an angle π / 4.

With the same success, we can explore arbitrary features of analytic functions.

Consider the flow with the potential *f (z),* a derivative *f '(z)* of which is the rational function, i.e. has only pole specifics $(z-z_0)^{-k}$. Then the function *f (z)* itself we can represent in the neighbourhood of a specific point in the form of

$$f(z) = A\ log\ (z - z_0) + A1\ (z - z_0)^{-1} + \ldots + \varphi\ (z) \tag{5}$$

where $\varphi(z)$ - function without specifics

Features of flows defined with the function $f(z)$ are made from specifics of streams made by individual components (5).

Let us consider an influence of the logarithmic term. Let us at first assume that A is a real number. Let us choose a circle of the radius r around the point z_0:

$z = z_0 + re^{i\varphi}$ *and assume that* $A \log(z - z_0) = u + i\upsilon$;

separating the real and imaginary parts, we obtain $A \log r = u$, $A\varphi = \upsilon$.

Streamlines $\upsilon = const$ will be radii going from the point z_0, while equipotential lines $u = const$ - will be circles with a centre in z_0

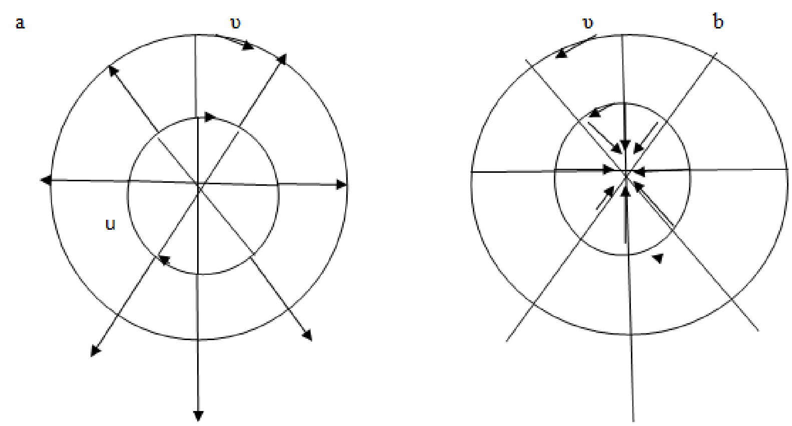

Figure 1: Divergent Flow or Drain, Source and Curls in the Near-Earth Environment.

Thus, the point z_0 will be either the source (*Fig.1* a) or the fluid outlet (*Fig.1* b), depending on the operator A (the liquid will either outflow, or flow into the point z_0). If A is a purely imaginary value, then we obtain the conjugate stream $A = iB$, $u = -B\varphi$, $\upsilon = \log r$. Circles will be streamlines. Such streams are called curls. Direction of motion (clockwise or counter clockwise) depends on the *B* operator.

We have obtained a great result. All features of the analytic function $f(z)$ on the sphere we can describe in terms of the fluid flow with a defined number of sources, outlets, curls, etc.[4].

3. MODELING OF THE BOSE-EINSTEIN CONDENSATE (QUANTUM FIFTH STATE OF MATTER) USING THE RIEMANN ZETA FUNCTION IN THE INTERPRETATION OF GRANT REMMEN

The hypothesis about the distribution of prime numbers was formulated by Riemann in 1859. But so far all attempts to prove it have ended in failure. This deceptively simple function has puzzled mathematicians since the 19th century. In this hypothesis, everything is tied to the Riemann zeta function ζ(s), which is an infinite series for a complex variable s, where $s = \sigma + it$.

$$\sum_{n=1}^{\infty} \frac{1}{n^s} = 1 + \frac{1}{1^s} + \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \cdots$$

The Riemann zeta function for n, where $s = \sigma + it$ is a complex number where σ and t are real numbers. This infinite series, called the Riemann zeta function ζ(s), is analytic (i.e., has definable values) for all complex numbers with real part greater than 1 *(Re(s) > 1)*. In this domain of definition, it converges absolutely. The Riemann conjecture states: "The non-trivial zeros of the Riemann zeta function ζ(s) have the real part (*Re(s) = 1/2)*". This is a modern formulation of an unproven assumption made by Riemann in his famous paper. It says that all points where zeta is equal to zero *(ζ(s) = 0)* on the critical strip $0 \le Re(s) \le 1$ have the real part $Re(s) = 1/2$. If this is true, then all non-trivial zeros of the zeta ζ(s) will be of the form $\zeta(1/2 + it)$. It seems that everything is quite simple, but neither mathematicians nor physicists can prove this hypothesis for more than 160 years. Initially, it was believed that the Riemann hypothesis is a purely mathematical fundamental problem that has no application in practice. But later it became clear that this problem is closely related to some aspects of quantum field theory. Essentially, quantum field theory is a set of tools that scientists can use to describe any set of interactions between particles. Solving the Riemann hypothesis (zeta function) will allow physicists to clarify the laws and better understand how our universe works. Perhaps the solution to this problem will also help open up physics beyond the Standard Model. There are many reasons to believe the Riemann hypothesis about the zeros of the zeta function is true. Perhaps the most compelling reason for mathematicians is the implications it will have for the distribution of primes. Numerical testing of the hypothesis at very high values suggests that it is true. In fact, the numerical confirmation of the hypothesis is so strong that in other fields, for example, in physics or chemistry, it could be considered experimentally proven. In a paper, an American physicist, the Fundamental Physics Fellow at the University of California, Santa Barbara Grant Remmen, reports on new approach to solving the Riemann problem. Grant Remmen is researching quantum field theory. While researching, he realized that one of the concepts in this theory has many characteristics in common with the Riemann zeta function [5]. So example, the scattering amplitude corresponds to the quantum mechanical probability that particles will interact with each other. The poles of the scattering amplitude correspond to the process of particle formation - a physical event that results in a particle with momentum. The value of each pole corresponds to the mass of the created particle. Specifically, a closed-form amplitude is constructed, describing the tree-level exchange of a tower with masses $m^2n = \mu^2n$, where $\zeta = 1/2 \pm i\mu n) = 0$. Requiring real masses corresponds to the Riemann hypothesis, locality of the closed-form amplitude to meromorphicity of the zeta function, and universal coupling between massive and massless states to simplicity of the zeros of ζ. Unitarity bounds from dispersion relations for the forward amplitude translate to positivity of the odd moments of the sequence of $1/\mu^2n$. Thus, we are talking about finding a function that behaves like a scattering amplitude and whose poles correspond to nontrivial zeros of the zeta function. The proof of the hypothesis about the zeros of the Riemann zeta function would make it possible to describe complex interactions during the birth and collision of elementary particles [5]. Grant Remmen managed to connect the mathematical properties of the Riemann zeta function with the physical Hilbert-Polya system, in which µn correspond to the eigenvalues of some quantum-mechanical Hamiltonian [6]. Dr.Grant Remmen has done a lot of work to try to find such an operator or to identify other connections to physics, including Dyson's observations. The resulting compact equation in terms of Landau-Riemann was illustrated by Dr. Grant Remmen in the article in Figure 2 [5].

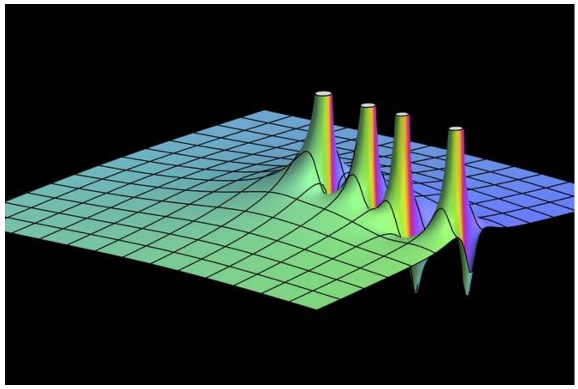

Figure 2: Illustration of Remmen's Formula for Amplitude in the Riemann Zeta Function.

At the end of the paper, Grant Remmen writes: "The universality property of the zeta function and its amplitude implications are worthy of further study. We leave these questions for future research." [5].

The quantum vacuum (dark matter) is, by definition, in a lower energy state than baryonic matter. The behavior of dark matter in such an energy state is similar to the behavior of atoms in a Bose-Einstein condensate (the quantum fifth state of matter), obtained at a matter temperature close to absolute zero - 273.5 Celsius or 0 Kelvin (Figure 3)

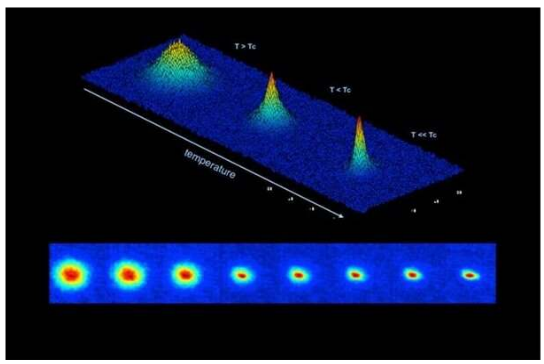

Figure 3: Bose-Einstein Condensate (BECS).

In June 2020, the Bose-Einstein condensate was successfully recreated in Earth orbit on the International Space Station (ISS) [7]. Only there it was possible to create all the conditions for the appearance of the quantum fifth state of matter within a few seconds, but this turned out to be enough for scientists to get an idea of how exactly dark matter moves and why we cannot see and feel it. Today, physicists say that instead of studying empty space, they can create a Bose-Einstein condensate and study the quantum vacuum [8]. An analytical interpretation of the Riemann zeta function by Professor Grant Remmen, which behaves like a scattering amplitude and whose poles correspond to the non-trivial zeros of the Bose-Einstein condensate (compare Fig. 2 with Fig. 3), would make it possible to transfer the Riemann hypothesis from the field of mathematics to the field of quantum physics.

4. CONCLUSIONS

Thus, the works of Riemann, written in the 19th century, turned out to be in demand in cosmology and quantum physics of the 21st century. The analytical functions and logical structures proposed by Riemann make it possible today to model the scalar fields of quantum gravity, the behavior of the physical vacuum in outer space, and the electrification of the Earth. In a curved Riemannian space-time, operating with the components of the 5-dimensional metric tensor, one can obtain ten components of the metric tensor of Einstein's general theory of relativity, four components of the electromagnetic vector potential Ā of Maxwell's electrodynamics, and one component that, in principle, can describe some new scalar field [9].

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